

Long-range acceleration induced by a scalar field external to gravity and the indication from Pioneer 10/11, Galileo and Ulysses Data

J.P. Mbelek and M. Lachièze-Rey
Service d'Astrophysique, C.E. Saclay
F-91191 Gif-sur-Yvette Cedex, France

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Abstract

The anomalous acceleration a_P of the Pioneer 10/11 - the Pioneer effect - has remained unexplained. We suggest an explanation based on the interaction of the spacecraft with a long-range scalar field, ϕ . The scalar field under consideration is external to gravity, coupled to the ordinary matter and undergoes obedience to the equivalence principle. In addition to its self-interaction term, the field is determined by an external source term proportional to the Newtonian potential in the weak fields limit it result a long-range acceleration a_P , asymptotically constant within the region of the solar system hitherto crossed by the spacecraft. Also, the limit $0.1 \cdot 10^{-8} \text{ cm/s}^2$, which follows from the Viking ranging data, is satisfied in the region of the terrestrial planets (in particular at the positions of the Earth and Mars), for a ϕ -field of mass $m_\phi \geq 1.8 \cdot 10^{-17} \text{ eV}/c^2$. The proposed solution gives the correct order of magnitude for a_P , as observed so far, and predicts the decline of a_P in the form of damped oscillations beyond 97 AU. An estimate of the cosmological constant is also made by taking into account the contribution of the vacuum energy density of the scalar field in galactic dynamics and particularly in the outskirts of the dwarf galaxy DDO 154.

1 Introduction

Recently, results from an almost twenty years study of radio metric data from Pioneer 10/11, Galileo and Ulysses spacecraft have been published by a team of the

NASA (Anderson et al. [1]), indicating an apparent anomalous, constant, acceleration acting on the spacecraft with a magnitude of the order $8.5 \cdot 10^{-8} \text{ cm/s}^2$, directed towards the Sun, to within the accuracy of the Pioneers' antennas and a steady frequency drift, a "clock acceleration", of about $-6 \cdot 10^{-9} \text{ Hz/s}$. A number of potential causes have been ruled out by the authors, namely gravity from Kuiper belt, gravity from the Galaxy, spacecraft "gas leaks", anisotropic heat (coming from the RTGs) reflection off of the back of the spacecraft high-gain antennae (Katz's proposal [2], see Anderson et al. [3]), radiation of the power of the main-bus electrical systems from the rear of the craft (Murphy's proposal [4], see Anderson et al. [5]), errors in the planetary ephemeris, and errors in the accepted values of the Earth's orientation, precession, and nutation, as well as nongravitational effects such as solar radiation pressure, precessional attitude-control maneuvers and a possible nonisotropic thermal radiation due to the Pu^{238} radioactive thermal generators. Indeed, according to the authors, none of these effects explain the apparent acceleration and some are 3 orders of magnitude or more too small, so they conclude that there is an unmodeled acceleration towards the Sun of $(8.09 \pm 0.20)10^{-8} \text{ cm/s}^2$ for Pioneer 10, $(8.56 \pm 0.15)10^{-8} \text{ cm/s}^2$ for Pioneer 11, $(12 \pm 3)10^{-8} \text{ cm/s}^2$ for Ulysses and $(8 \pm 3)10^{-8} \text{ cm/s}^2$ for Galileo. The authors plan to utilize two different transmission frequencies in further analysis to give an answer to whether there is some unknown interaction of the radio signals with the solar wind.

Since no "standard physics" plausible explanations for the residual acceleration has been found so far, the authors considered the possibility that the origin of the anomalous signal is the effect of a modification of gravity, for instance by adding a Yukawa force to the Newtonian or Milgrom's proposed modification of gravity (Milgrom [6]). They concluded however that neither easily works.

If the cause is dark matter, the amount needed to be consistent with the accuracy of the ephemeris should be only of order a few times $10^{-6} M_{\odot}$ even within the orbit

of Uranus (Anderson et al. [7]). Above all, the authors point out that the residual acceleration is too large to have remained undetected in the planetary orbits of the Earth and Mars. Indeed, the Viking ranging data limit any unmodeled radial acceleration acting on the Earth and Mars to no more than $0.1 \cdot 10^{-8} \text{ cm/s}^2$. Because of this severe constraint, the authors argue that, if the anomalous radial acceleration is of gravitational origin, it probably violates the principle of equivalence. But, an alternative is that the anomalous acceleration is asymptotically constant, rather than constant at all radii from the center of the solar system to the present location of Pioneer 10/11 spacecraft.

In this paper we propose an alternative explanation, based on the possible existence of a long range (non gravitational) scalar field, ϕ , which respects the (weak) equivalence principle. This possibility was previously introduced by Mbelek [8], to account for the rotational curves of spiral galaxies, as an alternative for dark matter. It gives, for the Pioneer 10/11 spacecraft, the correct order of magnitude for both the anomalous acceleration, a_P , and the clock acceleration, a_t . It proves to remain consistent with the planetary orbits determined from the Viking data.

As for the ordinary matter, the ϕ -field is a gravitational source through its energy-momentum tensor. A forthcoming paper will present the fundamental symmetry that may support it, from the background of classical fields theory. The plan of this paper is as follows : in section 2, we set the ϕ -field equation. Then, after linearization we divide space in three characteristic regions and find approximate exterior solutions for a static spherically symmetric source, actually the Sun. In section 3, Einstein equations are solved in the weak fields approximation to account for the metric tensor in the presence of the ϕ -field (out of the Sun). In section 4, the equation of motion of a test body in the presence of the ϕ -field is established. Solutions are found in the weak fields and low velocity limit. Then the anomalous long-range acceleration a_P is derived for the different regions of space. In section 5,

an interpretation of the data is proposed. In section 6, the steady frequency drift a_t is derived by using the equivalence principle. We finally conclude by an estimation of the cosmological constant by exploiting the declining part of the rotational curve (RC) of the dwarf galaxy DDO 154.

2 The scalar field equation

A manner to generate a constant radial acceleration could result from the introduction of a linear potential term in the Lagrangian of a test particle. An example is provided by the exterior solution of the locally conformal invariant Weyl gravity for a static, spherically symmetric source (Mannheim and Kazanas [9]). Unfortunately, Perlick and Xu [10], by matching the exterior solution to an interior one that satisfies the weak energy condition and a regularity condition at the center, show that this leads to contradiction of Mannheim and Kazanas's suggestion. They conclude that the conformal Weyl gravity is not able to give a viable model of the solar system.

This paper presents an alternative solution, under the form of a real scalar field, external to gravity but which satisfies the equivalence principle. We show below that it leads to the desired " Pioneer effect ", although it does not modify, as required, the orbital properties of the internal planets. The field ϕ obeys the equation

$$\nabla_\nu \nabla^\nu \phi = -U'(\phi) - J, \quad (1)$$

where the symbol ∇_ν stands for the covariant derivative compatible with the Levi-Civita connection. Equation (1) may be derived from Einstein equations provided that the energy-momentum tensor of the ϕ -field is of the form $T_{\mu\nu}^{(\phi)} = \partial_\mu \phi \partial_\nu \phi - g_{\mu\nu} [\frac{1}{2} \partial_\lambda \phi \partial^\lambda \phi - U(\phi) - \int J d\phi]$ (up to a positive multiplicative dimensionality constant, κ , for ϕ is dimensionless in this paper) and the energy-momentum tensor of the ordinary matter (matter or radiation other than the ϕ -field) is divergenceless (e.g., zero for the exterior solution and of the perfect fluid form for the interior solution).

In the rest of the paper, we apply the weak field approximation to the real classical scalar field ϕ and to the gravitational potentials, so that Newtonian physics apply. For a weak gravitational field, equ(1) above will write merely

$$\partial_\mu \partial^\mu \phi = -U'(\phi) - J. \quad (2)$$

The potential U denotes the self-interaction of ϕ , and we note $U'(\phi) = \frac{\partial U}{\partial \phi}$. The source term J , an external source function, takes gravity into account, as a source for the field ϕ . Of course, ϕ also acts as a source for gravity (through Einstein equations). We consider this latter action below in section 3.

In the weak field approximation, J depends on the Newtonian gravitational potential V_N , the only relevant scalar quantity related to a weak gravitational field. Thus, we write at first order $J = J(\frac{V_N}{c^2}) \approx -\frac{V_N}{r_0^2 c^2}$, where the constant r_0 defines a characteristic length scale (see subsection 5.1.1 for an estimation of r_0). The minus sign comes from the requirement that the effect of ϕ is similar to that of gravitation, so that $\frac{d\phi}{dr}$ and $\frac{dV_N}{dr}$ have the same sign (as we will see, the ϕ -field generates an acceleration term $\frac{d\phi}{dr}$, up to a positive multiplicative factor). This is in accordance with our previous study on the RC of spiral galaxies in which we found the positivity of $\frac{d\phi}{dr}$ necessary for the ϕ -field mimics a great part of the missing mass [8]. In the solar system, $\frac{d\phi}{dr}$ remains positive. The scalar field ϕ is positive definite throughout this paper.

Here we explore the effect of ϕ in the solar system, i. e., in the potential $V_N = -c^2 r_s/2r$ created by the static central mass of the the Sun, r being the radius from the centre (we choose as usual a zero value of the Newtonian potential at infinity), and r_s the Schwarzschild radius of the Sun. The problem has spherical symmetry, so that equation (2) yields finally

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} = U'(\phi) + \frac{r_s}{2r r_0^2}. \quad (3)$$

We will calculate the resulting ϕ -field and show that it creates an asymptotically constant acceleration: we solve the equation with the limiting condition (imposed by the weak fields approximation) that the field ϕ and its derivative $\frac{d\phi}{dr}$ are bounded for any given region of space. In addition, the field ϕ must vanish (up to an additive constant) if one sets $M = 0$, since the central mass M is its source ; the same condition applies to $\frac{d\phi}{dr}$.

As a first step to resolve equ(3), let us neglect for the moment the contribution of the self-interaction. The solution is

$$\phi = C + \frac{r_s}{4r_0^2}r - \frac{A}{r} \quad (4)$$

and thence,

$$\frac{d\phi}{dr} = \frac{r_s}{4r_0^2} + \frac{A}{r^2}, \quad (5)$$

where A and C are constants of integration. The constant of integration A of the dimension of a length obviously depends on r_s since the central mass is the source of the ϕ -field. Accordingly, we may set $A = \zeta r_s/2$, where ζ is a positive dimensionless constant that we will assume hereafter of the order unity. The positivity of ζ is inferred by the positivity of the spatial derivative $\frac{d\phi}{dr}$ at any distance from the centre. Note that the particular value $\zeta = 1$ involves the identity of the potential term $-A/r$ with the Newtonian one V_N/c^2 . We will show that, in a certain radius range and at sufficiently large distances from the centre, this represents the true solution to equ(3): the radial acceleration induced by the ϕ -field (neglecting its self-interaction), proportional to $\frac{d\phi}{dr}$, remains asymptotically constant. This will be referred to as the "Pioneer effect" throughout.

In order to solve the complete equation, we need to know the form of $U(\phi)$, although we will see that many results remain independent of this choice. To illustrate, we choose here a quartic self-interaction potential $U = U(0) + \frac{1}{2}\mu^2\phi^2 + \frac{\sigma}{4}\phi^4$, where $\sigma < 0$ is the self-coupling coefficient of the scalar field ; $\frac{1}{2}\mu^2\phi^2$ is the "mechanical"

mass term with $\mu = \frac{m_\phi c}{\hbar}$, m_ϕ denoting the mass of the scalar field. The reason for choosing a quartic polynomial form is that the corresponding quantum field theory should be renormalizable (Madore [11]).

This potential presents two extrema : one minimum at $\phi_I = \phi(r_I)$, with $U'(\phi_I) = 0$ and $U''(\phi_I) > 0$; and one maximum at $\phi_{III} = \phi(r_{III}) = \mu/\sqrt{|\sigma|}$, with $U'(\phi_{III}) = 0$ and $U''(\phi_{III}) < 0$ ($U''(\phi) = \frac{\partial^2 U}{\partial \phi^2}$). There is also an inflexion point at $\phi_{II} = \phi(r_{II}) = \mu/\sqrt{3|\sigma|}$, with $U'(\phi_{II}) > 0$ but $U''(\phi_{II}) = 0$. Since ϕ increases monotonically with respect to r , this corresponds to three regions I, II, III in space with $r_I < r_{II} < r_{III}$. Moreover, the monotony of ϕ together with the relation $\phi_{III} = \sqrt{3} \phi_{II}$ that links $\phi(r_{III})$ to $\phi(r_{II})$ involves (using the solution (4), in the first approximation) :

$$r_{III} \geq \sqrt{3} r_{II} \quad (6)$$

Let us call ϕ_0 , generically, a local extremum of $U(\phi)$ or $U'(\phi)$. In the neighbour region of space, $|\phi - \phi_0| \ll 1$, equation (3) may be solved in the weak field approximation by linearizing the function $U'(\phi)$ about ϕ_0 . This yields

$$\frac{d^2 \phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} - U'(\phi_0) - U''(\phi_0)(\phi - \phi_0) = \frac{r_s}{2r_0^2 r} \quad (7)$$

- in the first region of space (region I), $\phi_0 = \phi_I$
- in region II, $\phi_0 = \phi_{II}$
- in region III, $\phi_0 = \phi_{III}$.

Besides, as for the Higgs mechanism of symmetry breaking which too involves a scalar field and a quartic self-interaction potential, an analogy can be made with a well known phenomenon in solid state physics : the Meissner effect which is a phase transition of the second kind, between the superconducting to the normal state. Here, the field ϕ plays the role of the magnetic flux and $U''(\phi)$ plays the role of the difference $\Delta T = T_c - T$ between the temperature, T , of the solid and its critical

temperature, T_c .

- In region I, equ(7) reads

$$\frac{d^2\phi}{dr^2} + \frac{2}{r} \frac{d\phi}{dr} - \mu^2 (\phi - \phi_I) = \frac{r_s}{2r r_0^2}, \quad (8)$$

with solution

$$\phi = \phi_I - \frac{\bar{\lambda}^2}{2r_0^2} \frac{r_s}{r} (1 - e^{-(r-r_I)/\bar{\lambda}}) \quad (9)$$

which implies

$$\frac{d\phi}{dr} = \frac{\bar{\lambda}^2}{2r_0^2} \frac{r_s}{r^2} [1 - (1 + \frac{r}{\bar{\lambda}}) e^{-(r-r_I)/\bar{\lambda}}], \quad (10)$$

where $\bar{\lambda} = 1/\mu$ characterizes the dynamical range of the ϕ -field in region I. Clearly, it is necessary that $r_I = 0$ for the solution (9) be consistent with the conditions on ϕ and $\frac{d\phi}{dr}$.

Let us notice that, for a sufficiently massive ϕ -field ($m_\phi \geq 10\sqrt{2} \hbar/cr_\odot = 1.8 \cdot 10^{-17} \text{ eV}/c^2$, where r_\odot denotes the radius of the central mass), $\frac{d\phi}{dr}$ is smaller by a factor 1/100 than the same quantity that would be involved by relation (5). This means that the ϕ -field is expelled out from the central region I, so that the Pioneer effect is destroyed here.

This situation appears analogous to the Meissner effect where the magnetic flux is expelled out in the superconducting state ($\Delta T > 0$). That the ϕ -field is expelled from region I ($U''(\phi) > 0$), grants that the orbits of the internal planets are not modified. Its significant action on matter is restricted to regions II and III. Figure 1 shows the predicted curve $y = a_P/a_P^\infty$ versus $x = r/\bar{\lambda}$ for region I.

- In region II and about r_{II} , an approximate solution ϕ° is obtained by solving equation (7) on account that $\phi_0 = \phi_{II}$; one finds :

$$\phi^\circ = C + \frac{r_s}{4r_0^2} r - \frac{A}{r} + \frac{U'(\phi_{II})}{6} r^2 \quad (11)$$

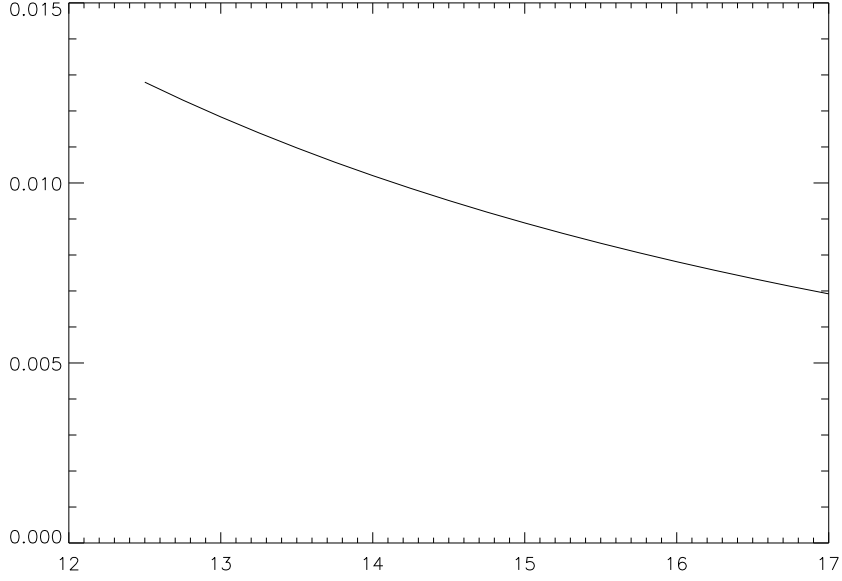


Figure 1: Predicted curve $y = a_P/a_P^\infty$ versus $x = r/\bar{\lambda}$ for region I. As one can see, the limiting condition $y \leq 0.01$ is satisfied for $x \geq 10\sqrt{2}$.

and

$$\frac{d\phi^\circ}{dr} = \frac{r_s}{4r_0^2} + \frac{A}{r^2} + \frac{U'(\phi_{II})}{3}r, \quad (12)$$

where A and C are constants of integration. Clearly, the extra potential $U'(\phi_{II})r^2/6$ will behave like a positive cosmological constant type term. Let us notice that the condition of the weak field approximation, $|\phi - \phi_{II}| \ll 1$, is always satisfied as long as $|r - r_{II}| \ll r_0^2/r_s$ in as much as the Λ term is neglected.

In region II below or beyond r_{II} , an improved solution $\phi = \phi^\circ + \delta\phi$ is obtained in the first approximation by adding to the previous solution ϕ° a correction term $\delta\phi$. This involves :

$$\frac{d^2\delta\phi}{dr^2} + \frac{2}{r}\frac{d\delta\phi}{dr} - U''(\phi^\circ)\delta\phi = 0 \quad (13)$$

Now, the "curvature" $U''(\phi)$, and hence its mean value, is positive between

r_I and r_{II} but negative between r_{II} and r_{III} . In the first approximation, one may write $U''(\phi < \phi_{II}) \approx U''(\phi_I) = 1/\bar{\lambda}^2$ and $U''(\phi > \phi_{II}) \approx -k^2$, with k a positive constant. We show in the following that $\lambda = \frac{2\pi}{k}$ defines a wavelength related to the part of region II beyond r_{II} . So, replacing $U''(\phi^\circ)$ by the value $-k^2$, equation (13) becomes in the first order approximation :

$$\frac{d^2\delta\phi}{dr^2} + \frac{2}{r}\frac{d\delta\phi}{dr} + k^2\delta\phi = 0. \quad (14)$$

The solution of the above equation is of the form

$$\delta\phi = \frac{B}{r} \sin(kr - \Phi_{II}), \quad (15)$$

where B is a constant of integration and Φ_{II} is a phase offset. Consequently, on account of the solution (11) and the continuity of ϕ at the radius r_{II} , the first order solution of equation (3) writes in region II beyond r_{II} :

$$\phi = C + \frac{r_s}{4r_0^2}r - \frac{A}{r} + \frac{U'(\phi_{II})}{6}r^2 + \frac{B}{r} \sin k(r - r_{II}), \quad (16)$$

$$\frac{d\phi}{dr} = \frac{r_s}{4r_0^2} + \frac{A}{r^2} + \frac{B}{r^2} \left[\frac{2\pi r}{\lambda} \cos\left(\frac{2\pi(r - r_{II})}{\lambda}\right) - \sin\left(\frac{2\pi(r - r_{II})}{\lambda}\right) \right] + \frac{U'(\phi_{II})}{3}r. \quad (17)$$

Below r_{II} , the solution is of the form :

$$\phi = \phi_I - \frac{\bar{\lambda}^2}{2r_0^2} \frac{r_s}{r} (1 - e^{-r/\bar{\lambda}}) + C + \frac{r_s}{4r_0^2}r - \frac{A}{r} + \frac{U'(\phi_{II})}{6}r^2 \quad (18)$$

The continuity of ϕ at the radius r_{II} involves :

$$\phi_I = \frac{\bar{\lambda}^2}{2r_0^2} \frac{r_s}{r_{II}} (1 - e^{-r_{II}/\bar{\lambda}}) \quad (19)$$

Further, the above solution involves a critical radius r_c at which the solutions of both regions I and II are connected. This critical radius is a solution of the following equation :

$$C + \frac{r_s}{4r_0^2}r - \frac{A}{r} + \frac{U'(\phi_{II})}{6}r^2 = 0. \quad (20)$$

The constant C is determined from relation (20) by requiring that r_s and A vanish whenever one sets M equal to zero. One finds $C = -U'(\phi_{II})/6 r_c^2$ and therefore :

$$r_c = \sqrt{2\zeta} r_0 \quad (21)$$

We will neglect throughout the contribution of cosmological constant type terms to the dynamics of the ordinary matter at the scale of the solar system since this is known to be very small at present epoch.

- In region III, the solution is of the form :

$$\phi = \phi_{III} + \frac{D}{r} \left\{ 1 - \cos \left[\frac{2\pi}{\lambda'} (r - r_{III}) \right] \right\} \quad (22)$$

which implies

$$\frac{d\phi}{dr} = -\frac{D}{r^2} \left\{ 1 - \cos \left[\frac{2\pi}{\lambda'} (r - r_{III}) \right] - \frac{2\pi r}{\lambda'} \sin \left[\frac{2\pi}{\lambda'} (r - r_{III}) \right] \right\}, \quad (23)$$

where D is a constant of integration and $\lambda' = 2\pi/\sqrt{|U''(\phi_{III})|}$ defines a wavelength for the ϕ -field in region III. Hence, *the ϕ -field would have a damped oscillatory behavior in the regions of space where $U''(\phi) < 0$.*

3 Einstein equations

3.1 The gravitational field sources

The metric tensor $g_{\mu\nu}$ is solution of the Einstein equations

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}. \quad (24)$$

In the presence of the scalar field ϕ , its right-hand side

$$T_{\mu\nu} = T_{\mu\nu}^{\circ} + T_{\mu\nu}^{(\phi)} \quad (25)$$

incorporates the energy-momentum tensor of the ordinary matter, $T_{\mu\nu}^{\circ}$, and the energy-momentum tensor of the ϕ -field itself. Let us emphasize that the scalar field

considered in this paper is external to gravity (like the electromagnetic field) but obeys the equivalence principle (unlike the electromagnetic field).

3.2 The weak fields approximation

Let us denote as $g_{\mu\nu}^\circ$ (resp. $g^{\circ\mu\nu}$) the solution of the Einstein equations for $\phi = 0$ and $g_{\mu\nu}$ (resp. $g^{\mu\nu}$) the components of the metric tensor in the presence of the ϕ -field (all greek indices run over 0, 1, 2, 3 and $x^0 = ct$) ; $R_{\mu\nu}$ denotes the Ricci tensor, $R = g^{\mu\nu} R_{\mu\nu}$ is the curvature scalar (Einstein's summation convention is adopted throughout this paper) and the $\Gamma_{\alpha\beta}^{\mu's}$ are the Christoffel symbols. Hereafter, whenever we assume spherical symmetry : $x^1 = r$, $x^2 = \theta$, $x^3 = \varphi$ (for the sake of simplicity, for planar motion $\varphi = \frac{\pi}{2}$ in the following), otherwise the $x^{i's}$ denote the Cartesian coordinates ($i = 1, 2, 3$). Einstein equations rewrite

$$R_{\mu\nu} = \frac{8\pi G}{c^4} [(T_{\mu\nu}^\circ - \frac{1}{2}T^\circ g_{\mu\nu}) + (T_{\mu\nu}^{(\phi)} - \frac{1}{2}T^{(\phi)} g_{\mu\nu})], \quad (26)$$

where $T^\circ = g^{\alpha\beta} T_{\alpha\beta}^\circ$ is the trace of $T_{\mu\nu}^\circ$ and $T^{(\phi)} = g^{\alpha\beta} T_{\alpha\beta}^{(\phi)}$ is the trace of $T_{\mu\nu}^{(\phi)}$. In the weak field approximation, one gets in particular : $T_{00}^{(\phi)} - \frac{1}{2}T^{(\phi)} g_{00} = -\kappa(U(\phi) + \int J d\phi)$ and $T_{00}^\circ - \frac{1}{2}T^\circ g_{00} = \frac{1}{2}\rho c^2$ (weak gravitational field approximation). Furthermore, one has in the first approximation

$$R_{00} = \frac{1}{2}\nabla^2 g_{00}. \quad (27)$$

So, we may write :

$$g_{00} = 1 + 2\frac{V_N - V_\phi}{c^2} \quad (28)$$

with

$$\nabla^2 V_\phi = \frac{8\pi G}{c^2} \kappa(U(\phi) + \int J d\phi) \quad (29)$$

$\nabla^2 V_N = 4\pi G\rho$ and $g_{00}^\circ = 1 + 2V_N/c^2$, where ρ is the density of the ordinary matter.

Derivating partially equ(29) with respect to ϕ then comparing with equ(2) yields :

$$\frac{\partial V_\phi}{\partial \phi} = (\frac{\partial V_\phi}{\partial \phi})_{r=r_I} + \frac{8\pi G}{c^2} \kappa\phi, \quad (30)$$

on account that the derivative $\frac{\partial V_\phi}{\partial \phi}$ should be bounded even when extrapolated at $r = 0$.

4 Equation of motion

The equation of motion of a test body in the presence of the scalar field ϕ writes in curved spacetime :

$$\frac{du^\mu}{ds} + \Gamma_{\alpha\beta}^\mu u^\alpha u^\beta = -\partial^\mu \phi + \frac{d\phi}{ds} u^\mu. \quad (31)$$

This means that a force term $F^\mu = mc^2[\partial^\mu \phi - (d\phi/ds)u^\mu]$ enters in the right-hand side of the equation of motion of a test body of mass m in the presence of the ϕ -field. The first term of the right-hand side, $\partial^\mu \phi$, is analogous to the electric part of the electromagnetic force whereas the second one $-(d\phi/ds) u^\mu$ is analogous to the magnetic part. Both terms are necessary to satisfy the unitarity of the velocity 4-vector ($u_\mu u^\mu = 1$, hence $u_\mu(u^\nu \nabla_\nu)u^\mu = 0$). Equation (31) may be derived from the Lagrangian :

$$L = -\frac{mc^2}{2} e^{-\phi} (g_{\mu\nu} u^\mu u^\nu + 1). \quad (32)$$

4.1 Motion in weak fields with low velocity

In the weak fields and low velocity limit, equation (31) simplifies to

$$\frac{d^2 x^i}{dt^2} = -c^2 \Gamma_{00}^i - g^{ii} \frac{\partial \phi}{\partial x^i} c^2 + \frac{d\phi}{dt} \frac{dx^i}{dt}, \quad (33)$$

Now, $g^{ii} \simeq -1$ and $\Gamma_{00}^i \simeq -1/2 g^{ii} \partial g_{00} / \partial x^i$. Hence, the equation of motion rewrites in vectorial notation :

$$\frac{d^2 \vec{r}}{dt^2} = -\vec{\nabla} V_N - f c^2 \vec{\nabla} \phi + \frac{d\phi}{dt} \frac{d\vec{r}}{dt} \quad (34)$$

where we have set $f = \frac{\partial(V_\phi/c^2)}{\partial \phi} - 1$. In next section 4.2, we show that f is positive.

4.2 Derivation of the long-range acceleration a_P

The projection of equation (34) above in plane polar coordinates (r, θ) yields, assuming $|dr/dt| \ll c\sqrt{f}$, the radial component of the acceleration vector,

$$a_r = -\frac{GM}{r^2} - f \frac{d(\phi c^2)}{dr}. \quad (35)$$

In the low velocity limit, the tangential component of the acceleration vector, a_θ , is equal to zero (conservation of the angular momentum). Clearly, relation (35) is of the form :

$$a_r = -(a_N + a_P) \quad (36)$$

where $a_N = \frac{GM}{r^2}$ is the magnitude of the Newtonian radial acceleration and a_P is the radial acceleration induced by the scalar field,

$$a_P = fc^2 \frac{d\phi}{dr}. \quad (37)$$

Relation (37) applies to any region of space out of the central mass. Besides, since $d\phi/dr > 0$, f must be positive for a_P mimics a missing mass gravitational field (see section 7 below). Hence, it follows that the radial acceleration induced by the scalar field will be directed towards the central mass as observed for Pioneer 10/11, Ulysses and Galileo.

4.2.1 Region I

In region I and for $r \gg \bar{\lambda}$, the scalar field is expelled out and consequently equation (34) simplifies to :

$$\frac{d^2 \vec{r}}{dt^2} = -(1 + f_0 \frac{\bar{\lambda}^2}{r_0^2}) \vec{\nabla} V_N \quad (38)$$

or equivalently

$$\frac{d^2 \vec{r}}{dt^2} = -G \frac{M + M_{hidden}}{r^2} \vec{u}_r, \quad (39)$$

where $f_0 = f(0)$, $\vec{u}_r = \vec{r}/r$ is the radial unitary vector and $M_{hidden} = f_0(\bar{\lambda}/r_0)^2 M$ mimics a hidden mass term (so that the true dynamical mass of the Sun differs from

its luminous mass, M_\odot , by the amount $f_0(\bar{\lambda}/r_0)^2 M_\odot$). It is worth noticing that the kind of missing mass which is invoked here mimics a spherical distribution of dark matter located within the Sun rather than a solar halo dark matter. In this respect, we will distinguish the hidden mass from dark matter. Throughout, hidden mass means extra terms involving the ϕ -field and that mimic a mass term. Both hidden mass and dark matter define the missing mass.

Furthermore, since the ϕ -field respects the equivalence principle, equation (38) involves a maximum shift Z on the frequency of a photon given by :

$$Z = (1 + f_0 \frac{\bar{\lambda}^2}{r_0^2}) \Delta V_N / c^2. \quad (40)$$

Equation (40) above allows us as yet to put an upper bound on the possible value of f_0 . Indeed, analysis of the data from the tests of local position invariance ("the outcome of any local non-gravitational experiment is independent of where and when in the universe it is performed") yields a limit $f_0 < 2 \cdot 10^{-4} r_0^2 / \bar{\lambda}^2$ (see C. M. Will [14]). The local position invariance is one of the three pieces of the equivalence principle and, since the ϕ -field respects the equivalence principle, this is a crucial test for this field. As we will see further, the ϕ -field passes the current tests. Indeed, one finds that f_0 is of the order 10^{-6} (see subsection 5.1.2). In addition, the study of the possible effect of dark matter on the motion of the outer planets involves that the missing mass within the Sun is necessarily less than $10^{-6} M_\odot$ (see Anderson et al. [7]). Thence, we may conclude that $\bar{\lambda} \ll r_0$. Clearly, a ϕ -field of mass $m_\phi \geq 1.8 \cdot 10^{-17} \text{ eV}/c^2$ passes all the current tests.

4.2.2 Region II

Let us neglect for the moment the contribution of the damped oscillations. We will also neglect the ϕ -term in relation (30) so that $f \simeq f_0$ (i.e., we neglect the anharmonic terms). Replacing $\frac{d\phi}{dr}$ by the expression (5) obtained for region II,

relation (37) yields :

$$a_P = a_P^\infty \left(1 + 2 \frac{r'_0{}^2}{r^2}\right) \quad (41)$$

where we have set

$$r'_0 = \sqrt{\zeta} r_0 \quad (42)$$

and

$$a_P^\infty = \frac{f_0}{2} \frac{GM}{r_0^2} \quad (43)$$

turns out to be the asymptotic radial residual acceleration. Besides, combining relations (21) and (42) above yields

$$r_c = \sqrt{2} r'_0 \quad (44)$$

and thence

$$a_P(r = r_c) = 2a_P^\infty \quad (45)$$

which is also the maximum possible value for a_P . Relation (44) may also be derived by requiring the continuity of the derivative $d\phi/dr$ at radius r_c (neglecting the Λ term).

5 Interpretation of the data

It has been questioned why the Pioneer effect has gone undetected in the planetary orbits of the Earth and Mars. Precisely, the Viking ranging data limit any unmodeled radial acceleration acting on Earth and Mars to no more than $0.1 \cdot 10^{-8}$ cm/s². Indeed, since the Pioneer effect is expected in region II but not in region I, there must be some critical radius r_c which allows one to distinguish between these two region of space within the solar system. As region II is defined about the radius r_{II} , one may reasonably consider that r_c is of the order $r_{II}/10$ and accordingly the anomalous acceleration a_P should be negligibly small below the radius $r_c/2$. Indeed, our estimate of r_{II} , given in subsection 5.2, is in accordance with the estimate of

r_c given in subsection 5.1.1 and both estimations corroborate the fact that the Pioneer effect is negligibly small below the asteroid belt. Our scalar field, external to gravity but which respects the equivalence principle, provides a solution to both the anomalous radial acceleration observed on the spacecraft and the absence of a comparable effect on the Earth or Mars.

Indeed, it is worth noticing that all the spacecraft which undergo the Pioneer effect were located at radii well beyond the orbital radius of Mars when the data were received from them (the closest spacecraft, Galileo and Ulysses, were in the vicinity of Jupiter).

Moreover, "no magnitude variation of a_p with distance was found, within a sensitivity of $2 \cdot 10^{-8} \text{ cm/s}^2$ over a range of 40 to 60 AU". On account of these facts, we conclude that the Pioneer effect is a distance effect and a_p is rather asymptotically constant within the regions hitherto crossed by the spacecraft. Above all, the scalar field approach leads to the same conclusion.

5.1 Estimate of a_P for Pioneer 10/11 using Ulysses data

To start with, let us recall that no magnitude variation of a_P with distance was found, within a sensitivity of $2 \cdot 10^{-8} \text{ cm/s}^2$ over a range of 40 to 60 AU (the data analysis of unmodeled accelerations began when Pioneer 10 was at 20 AU from the Sun). Thus we may set $a_P \approx a_P^\infty$ for the Pioneer 10/11. Since we need to be given at least one point in the curve a_P versus r to be able to determine all the parameters needed, our strategy will consist to use a piece of information from the Ulysses data (the nearest point) to compute the Pioneer 10/11 data (the farthest points). It is worth noticing that the piece of information considered by itself gives no information on the magnitude of the long-range acceleration of the spacecraft. It is this feature that makes the adopted procedure relevant. To compute a_P^∞ we need to estimate first r_0 and f_0 .

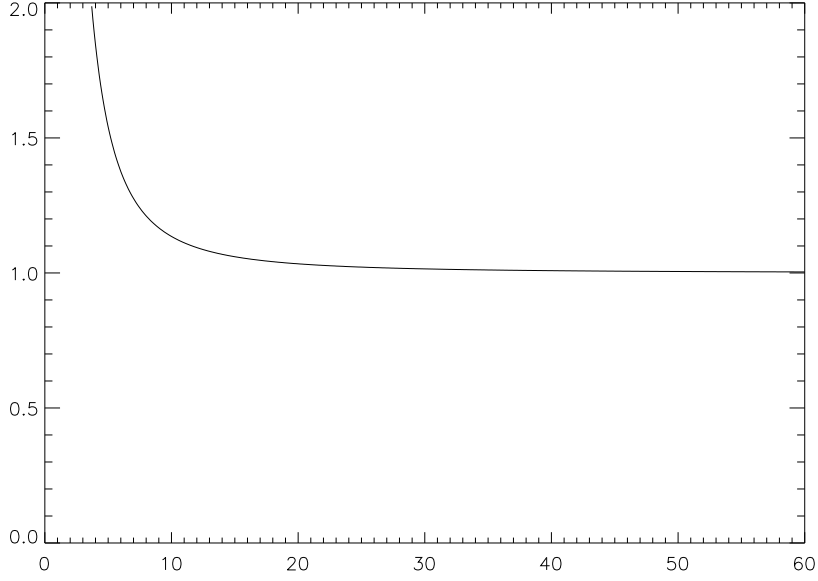


Figure 2: Predicted curve a_P/a_P^∞ versus the radius (in AU) from 3.9 AU to 60 AU. The curve is asymptotically flat between 40 AU and 60 AU (the damped oscillations expected beyond r_{II} have been neglected).

5.1.1 estimate of r_0 and r_c

As one can see, for $r = 2r'_0$, relation (41) implies that $a_P = \frac{3}{2}a_P^\infty$. Now, this was observed for Ulysses in its Jupiter-perihelion cruise out of the plane of the ecliptic (at 5.5 AU). This is also consistent with Galileo data (strongly correlated with the solar radiation pressure ; correlation coefficient equal to 0.99) if one adopts for the solar radiation pressure (directed away from the Sun) a bias contribution to a_P equal to $(-4 \pm 3) 10^{-8} \text{ cm/s}^2$. Hence, we conclude that r'_0 is approximately equal to half of Jupiter's orbital radius, that is $r'_0 \approx 2.75 \text{ AU}$ and consequently $r_c \approx 3.9 \text{ AU}$ on account of relation (44). Let us assume for the moment ζ equal to unity ; this leads to conclude that $r_0 \approx 2.75 \text{ AU}$. Figure 2 shows the shape predicted for the curve a_P/a_P^∞ versus the radius. The plot starts from the radius $r_c = 3.9 \text{ AU}$ to the radius $r = 60 \text{ AU}$.

5.1.2 estimate of f_0 and derivation of the magnitude of a_P^∞

In our study on the RC of spiral galaxies, we found that f_0 is of the order $\frac{v_{max}^2}{c^2}$, where v_{max} denotes the maximum rotational velocity. This seems to be a general order of magnitude for this parameter. So, in what follows, we derive an estimation of f_0 using the relation $f_0 \simeq \frac{v_{max}^2}{c^2}$, where v_{max} is a maximum velocity to be determined for the solar system. In the case of interest in this paper, the ϕ -field under consideration though external to gravity is generated from the Sun (or any other star we would have considered). Therefore, it seems natural that v_{max} should be a typical velocity that is related to the matter components of this star and not a peculiar orbital velocity. A suitable value for v_{max}^2 (see Ciufolini [17]), perhaps the best for it involves thermodynamics parameters solely, is given by the ratio P_c/ρ_c (assuming the perfect gas), where P_c and ρ_c denote respectively the central pressure and mass density of the star under consideration. Taking the value of T_c given by solar models (Stix [15], Brun et al. [16]), the expression $v_{max}^2 = \frac{P_c}{\rho_c}$ gives for the Sun : $f_0 = (1.72 \pm 0.04) 10^{-6}$ and $a_P^\infty = (6.8 \pm 0.2) 10^{-8} \text{ cm/s}^2$, in good agreement with the recent results which give $a_P^\infty = (7.29 \pm 0.17) 10^{-8} \text{ cm/s}^2$ as the most accurate measure of the anomalous acceleration of Pioneer 10 (Turyshchev et al. [12]). Further, the value computed for a_P^∞ may be corrected to $a_P^\infty = (7.23 \pm 0.2) 10^{-8} \text{ cm/s}^2$ by identifying λ with r_0 (see subsection 5.2 below) : hence, $\zeta \simeq 1.07$.

5.2 Damped oscillations and vanishing of a_P

Figure 1 of the paper of Anderson et al. shows an almost harmonic oscillation of a_P (nothing is said about this by the authors themselves though) for Pioneer 10 which starts at the radius $r_{II} = 56.7 \pm 0.8 \text{ AU}$ with an amplitude a_{Pm} of the order $\frac{1}{4} 10^{-8} \text{ cm s}^{-2}$ (this is derived by comparison with the uncertainty on a_P for Pioneer 10) and a wavelength $\lambda = 2.7 \pm 0.2 \text{ AU}$ the value of which turns out to be quite identical to that of r_0 (let us notice by passing that r_{II}/λ is an integer ($= 21$)). With these

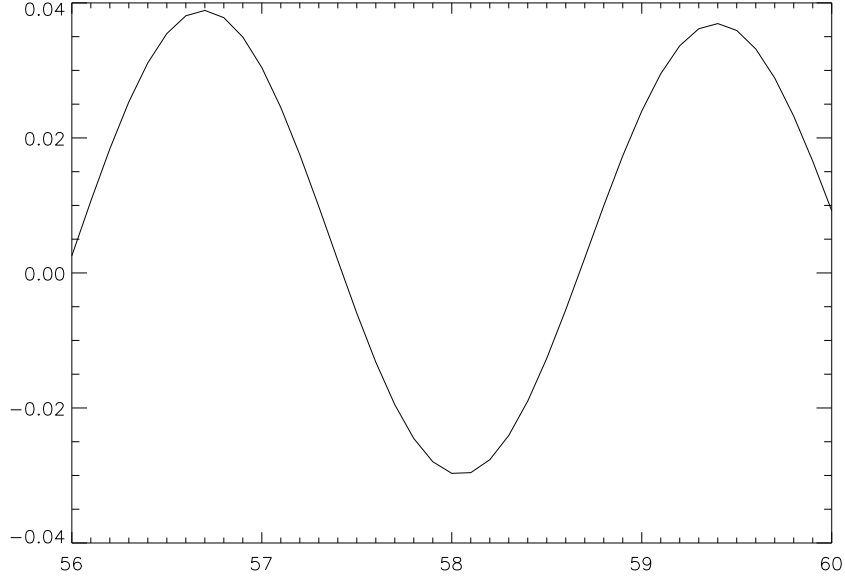


Figure 3: Predicted curve $(a_P - a_P^\infty)/a_P^\infty$ versus r (in AU), for region II with parameters $r_{II} = 56.7$ AU, $\lambda = 2.7$ AU, $\zeta = 1$ and $B = A/16$.

observational data, relation (17) involves, for $\zeta \simeq 1$ and $B = A/16$, $a_{Pm} \simeq 0.26 \cdot 10^{-8}$ cm s $^{-2}$ between 56 AU and 60 AU as can be seen in figure 3.

Furthermore, the calculations carried out in subsection 2 lead to predict the decline of a_P in the form of damped oscillations beyond $r = r_{III}$. Hence, since $r_{III} > \sqrt{3} r_{II}$, we may confidently expect the decline of a_P to occur only beyond $r = 96.8$ AU. Let us emphasize that the spatial periodicity λ involves a temporal periodicity $T_P = \lambda/v_P$, where v_P is the speed of the spacecraft. Hence, the periodicity of one year found for Pioneer 10 has nothing to do with the orbital periodicity of the Earth. Indeed, the coincidence just comes from the fact that $v_P = 2.66$ AU/yr for Pioneer 10 (at least since 1987). As a consequence, T_P should be greater than one year for Pioneer 11 since Pioneer 10 is faster moving than Pioneer 11.

6 Derivation of the steady frequency drift using the equivalence principle

Since, the ϕ -field obeys the equivalence principle, the steady frequency drift may be explained in another way than the Doppler effect thanks to this principle (cf. Misner et al [13]). Actually, the steady frequency drift and the corresponding "clock acceleration" $-a_t = -2.8 \cdot 10^{-18} s/s^2$ shown by the Compact High Accuracy Satellite Motion Program analysis of Pioneer 10 data may also be interpreted as the analogous of the gravitational redshift linked to the extra potential term $V_P = \int a_P dr$ associated to the scalar field. Indeed, the frequency drift $\frac{d\Delta\nu}{dt}$ as well as the clock acceleration $-a_t = \frac{d(\frac{\Delta\nu}{\nu})}{dt}$ follow from the relation $\frac{\Delta\nu}{\nu} = \frac{V_P(r_\oplus) - V_P(r)}{c^2} = -\frac{1}{c^2} \int_{r_\oplus}^{r_\oplus + ct} a_P dr$, where r_\oplus denotes the orbital radius of the Earth and $r = r_\oplus + ct$ (one way, as considered by the authors) is the distance of the spacecraft from the Earth. Therefore, on account that $dr = c dt$ for the photons (one way), one obtains the observed relation $a_P = a_t c$. In this way, the identity $a_P = a_t c$ seems more natural since, in this approach, it is indeed the photons received on Earth from the spacecraft that are concerned instead of the spacecraft themselves.

7 Conclusion

In this paper, we have presented a possible explanation of the "Pioneer effect", without being in conflict with the Viking data or the planetary ephemeris. This is based on a possible interaction of the spacecraft with a long-range scalar field, ϕ , which respects the equivalence principle. Like any other form of matter-energy, the ϕ -field is a gravitational source through its energy-momentum tensor. Conversely, its source is the Newtonian potential of the ordinary matter (in this case, the Sun). The calculations were performed in the weak fields approximation, with U a quartic self-interaction potential. They gave, near the spacecraft, a residual radial accel-

ation directed towards the Sun, with a magnitude a_P asymptotically constant (in region II). Both a_P and the corresponding clock acceleration, a_t , computed from our formulas are in fairly good agreement with the observed values. Moreover, a scalar field of mass $m_\phi \geq 1.8 \cdot 10^{-17} \text{ eV}/c^2$ will be expelled from region I in a way quite analogous to the Meisner effect in a superconducting medium. This limits a_P to no more than $0.1 \cdot 10^{-8} \text{ cm/s}^2$, from the radius of the Sun to $r_c = 3.9 \text{ AU}$. It is also found that the a_P term should be accompanied with damped oscillations in the intermediary region between region II and region III. We also predict beyond $r_{III} = 97 \text{ AU}$ (that is, about the year 2009 or so, for Pioneer 10) the vanishing of a_P in the form of damped oscillations.

Furthermore, the scalar field theory, as developped in this paper, also gives good fits for the rotational curves of spiral galaxies as shown in a previous study. Moreover, the same field acts at the cosmological scales like a cosmological constant. Preliminary estimations (Mbelek and Lachièze-Rey, in preparation) from the dynamics of the external region of the dwarf galaxy DDO 154, actually the sole galaxy for which the edge of the mass distribution has been reached (see Carignan & Purton [18]) led to a value $\Omega_\Lambda = 0.43 (\text{H}_0/100 \text{ km s}^{-1} \text{ Mpc}^{-1})^{-2}$ in fairly good agreement with the value $\Omega_\Lambda = 0.7$ deduced from the Hubble diagram of the high-redshift type Ia supernovae (Perlmutter et al. [19], Schmidt et al. [20], Riess et al. [21], Garnavich et al. [22]) for H_0 about $75 \text{ km s}^{-1}/\text{Mpc}$.

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